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RESEARCH MEMORANDUM

PERFORMANCE OF SUPERSONIC AXIAL-FLOW COMPRESSORS BASED ON ONE-DIMENSIONAL ANALYSIS

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Cleveland, Ohio

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

WASHINGTON

March 25, 1949

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PERFORMANCE OF SUPERSONIC AXIAL-FLOW COMPRESSORS BASED ON ONE-DIMENSIONAL ANALYSIS

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SUMMARY

An analytical study was made of four combinations of subsonic and supersonic flow in the rotor- and stator-blade passages of axial-flow compressors. The theoretical total-preeeure ratio of each of these types of compressor, when only normal-shock losses are considered, is given as a function of the rotational speed and the turning angle in the rotor passage.

Compressor I with a rotor-contained normal shock and subsonic velocity at the stator entrance Offered 8 total-pressure ratio Of about 3.5 per stage and for no change in outer shroud radius was limited by the stator-entrance angle8 and stator-entrance Mach numbers to 8 maximum total-pressure ratio of about 4.6 per stage.

Compressor II had a rotor-contained normal shock but permitted supersonic velocities at the stator entrance. Total-pressure ratios above 6.0 per stage were theoretically possible, but problems such a8 stabilizing normal shocks in both the rotor and the stator, large turning in the rotor passage, and utilizing the high stator-entrance angles require further investigation.

Compressor III utilized supersonic flow throughout the rotor and recovered the pressure in supersonic stators having a contained normal shock. Total-pressure ratios above 5.0 were possible in a single stage, but normal shock losses were greater than for the other types at comparable pressure ratios above 3.0. Because the shock was removed to the stator, however, devices for approaching isentropic deceleration through sonic velocity may be more easily applied.

Compressor IV had subsonic velocity in the rotor and supersonic velocity in the stator entrance. Theoretical total-preeeure ratios above 4.0 per stage were 'possible, but the range of efficient supersonic operation was expected to be from 1.8 to 2.2.

The specific ma88 flow Of compressors I, II, and III was above 90 percent of maximum for the annulus. The specific mass flow Of compressor IV was about 60 percent of maximum for the annulus; however, because of the possible lack of restrictions on radius ratio, compressor IV may have had a total mass flow comparable with other types of compressor for a given frontal area.

INTRODUCTION

The use of supersonic air velocities relative to any of the component8 of axial-flow compressors offers promise of greatly increasing the total-pressure ratio per stage (reference 1). Several of such compressors have been built and operated (references 2 and 3). The maximum theoretical performance attainable with various supersonic configurations of axial-flow compressors considered structurally and aerodynamically practicable at present was briefly analyzed at the NACA Lewis laboratory and is presented herein.

For the purpose of this discussion, a supersonic axial-flow compressor Can be considered as any axial-flow compressor having supersonic velocities relative to any component. The velocities at the entrance to the inlet guide vanes can be subsonic or supersonic. Because the rem energy can be recovered more efficiently by the use of spiked diffusers than by the compressor and because the mass flow per unit area decreases appreciably for Mach numbers above 1.2, only subsonic velocities have been considered at the entrance to the inlet guide vanes, even for the case of supersonic aircraft. The rotor or diffusing stators can have relative velocities that are: (1) supersonic at the entrance with deceleration through sonic velocity by means of shocks inside the passage, (2) entirely supersonic throughout the passage, or (3) subsonic throughout the passage. For most applicationa, the velocities leaving the exit eta-tore must be subsonic; therefore the use of supersonic velocities throughout the stators is not considered. Practicable supersonic compressor8 may thus tentatively be restricted to those having both subsonic entrance velocities into the inlet guide vanes and eubsonic exit velocities from the diffusing stators; six possible combinations of rotor and stator flows then remain. Subsonic flow throughout both rotor and stator is the usual subsonic compressor and is not considered. Supersonic flow throughout the entire rotor with subsonic stator velocities give8 total-pressure ratios below 1.6 for rotor speeds up to 1800 feet per second and hence this flow configuration is also disregarded. Thie analysis is therefore restricted to the remaining four types of compressor, which are described in the following table:

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Compressor	Velocity at rotor	Velocity at stator
I	Supersonic entrance, deceleration through sonic velocity in passage	Subsonic throughout
II	Supersonic entrance, deceleration through sonic velocity in passage	Supersonic entrance, deceleration through sonic velocity in passage
III	Supersonic flow throughout	Supersonic entrance, deceleration through sonic velocity in passage
IV	Subsonic flow throughout	Supersonic entrance, deceleration through sonic velocity in passage

GENERAL DESCRIPTION OF FLOW PROCESSES

Energy addition. - The energy addition in any compressor rotor appears as kinetic energy and static-pressure increase. The usual subsonic axial-flow compressor obtains static-pressure increase by diffusing the flow into the larger area made available by turning the flow in the direction of rotation. The supersonic compressor, however, can have a large increase in static pressure in a relatively short distance without requiring blade curvature by means of the compression shock. Any turning that can be accomplished in the direction of rotation adds to the absolute kinetic energy, further increasing the total energy addition.

Compressor I (rotor-contained normal shock, subsonic entry into stators) obtains most of its total-pressure ratio through the static-pressure gain in the rotor. A small static-pressure rise occurs when the supersonic stream entering the rotor passage is compressed to the minimum area that permits stable supersonic flow (reference 4). A large static-pressure rise occurs across the normal shock that takes place at the minimum-area Mach number under proper operating conditions. If the air can be further diffused in the subsonic portion of the blade passage, an additional static-pressure increase is obtained.

Compressor II (rotor-contained normal shock, supersonic stator entrance) has the static-pressure increase due to deceleration through sonic velocities in the rotor, as obtained in compressor I; and, in addition, imparts sufficient kinetic energy by turning and diffusing the flow in the rotor passage to cause the air to enter the stators with supersonic velocities. This kinetic energy is then converted to pressure energy by deceleration through sonic velocities in the stators. 1067

Compressor III (supersonic rotor throughout, supersonic stator entrance) obtains most of its total-pressure ratio through kinetic-energy addition. The total amount of contraction of the supersonic stream is limited by the contraction ratio that permits stable supersonic flow to enter the passage. The static-pressure increase obtained by compressing the supersonic stream is therefore limited. The kinetic energy is obtained by turning the supersonic stream until the air enters the stators with supersonic velocities. The stators then decelerate the air through sonic velocity, converting the kinetic energy to pressure energy,

Compressor IV (subsonic rotor, supersonic stator entrance) relies primarily on the addition of kinetic energy by turning the flow in the direction of rotation. A relatively small static-pressure rise through the rotor can be obtained by diffusing the flow at the same time that it is being turned in the rotor passage. Sufficient energy must be added so that the velocities entering the diffusing stators are supersonic. This energy is then converted to pressure energy in the stators by deceleration through sonic velocities and further diffusion to the desired velocity.

Entrance conditiona. - For the compressors requiring supersonic entrance velocities relative to the rotor (compressors I, II, and III), guide van88 may be necessary to increase the Mach number relative to the rotor by turning the flow opposite to the direction of rotation. This turning is moat essential at the blade root where the rotational speeds are lowest.

For compressors I and II, a large change in static pressure and tangential velocities occurs through the normal shock in the rotor. These rapid changes may lead to large detrimental radial accelerations immediately behind the shock unless controlled through varying the relative total-pressure conditions at the rotor entrance by use of entrance guide vanes. Compressore I and II therefore require entrance guide van88 for equilibrium considerations and compressor III may require guide vanes to increase the Mach number relative to the rotor-blade root.

For compressor IV, subsonic velocities relative to the rotor are required. For rotational speeds above 900 feet per second and axial Mach numbers above 0.20, the air must be turned in the direction of rotation in order to keep the relative velocities subsonic. The turning in the guide vanes required for subsonic velocities relative to the rotor restricts the maximum axial velocity that can be used for any given rotational speed. The maximum permissible axial component then decreases as the design rotational speed is increased.

Rotor flow. - The turning that is required in the rotor passage for compressors I and II can be accomplished either before or after the normal shock without influence cm the total-pressure ratio. However, the advantage8 to turning in the supersonic region are: (1) An increase in blade thickness is allowed; and (2) separation is less likely to occur because (a) the expansion caused by supersonic turning in the direction of rotation produces a decreasing pressure gradient, and (b) the turning is accomplished before the build-up in boundary layer usually accompanying a normal shock.

As the rotor passage is turned in the direction of rotation, the flow area increases for constant blade thickness. If the chord length remains approximately constant, the rate of area increase and consequently the subsonic diffusion rate becomes larger as the amount of turning is increased. Increased rotor-blade turning with constant chord may lead to diffusion rates that would cause separation unless the flow area were restricted by decreasing the blade height or span. A restriction of the blade height by changing the hub or tip radii is therefore probably necessary for compressors utilizing a large turning in the rotor passage (compressors II, III, and IV, and possibly compressor I).

Diffusing stator conditions. - Compressors I and II may have very high entrance angles into the diffusing stators. The axial component of the velocity is decreased considerably because of the large change in density in going through the shock. For the case in which the axial velocity becomes mall compared with the tangential velocity entering the stators, the stator entrance angles measured from the axis may become very large. The axial component of the velocity can be increased by restricting the rotor-exit area through change in hub or tip radius, thereby lowering the angle into the stator a8 well as decreasing the diffusion rate.

Because of the small change in density across the rotor of compressor III, the axial velocity decreases only slightly and the entrance angles into the stators are considerably lower than for compressors I and II.

Compressor IV Can have an increase in static pressure across the rotor as well as kinetic-energy addition, but because the pressure gradient caused by both the diffusion and the turning aggravates boundary-layer build-up, it may be advantageous to restrict the diffusion to little or none. When the diffusion is so restricted, little or no change in density across the rotor exists and the entrance angles into the Stators are not as high as for compressors I or II. 1067

Velocities entering the stators near a Mach number of 1.0 are probably difficult to handle. For subsonic Mach numbers near 1.0, the blade critical Much number cannot be far exceeded. For supersonic Mach numbers very close to 1.0, the angle through which the flow can be deflected without forming a detached shock is very Small, thus stator blades with very small wedge angles will be required. Also, if the flow is only slightly off design, the angle of incidence plus the wedge angle may be sufficient to cause formation of a detached shock or bow wave.

Efficiency. - The only losses considered in the computations were the losses due to the normal shook. If a method of approaching isentropic deceleration through the velocity of sound can be adapted, however, these 108888 can be largely eliminated. The possibility of adapting devices to approach isentropic deceleration exists for all cases in which the shock occurs, although the use of such devices on the rotating passage may prove difficult.

For pressure ratios above 3.0, the added energy is obtained largely through kinetic-energy addition. Higher solidities are therefore required to accomplish the desired turning in the rotor passage. Also, the added kinetic energy increases the flow angle into the stators, possibly necessitating extra stator rows. The viscous loss then probably increases with increased pressure ratios.

PERFORMANCE

This analysis is based on equations for one-dimensional flow, where the Mach number is a function of only the passage area normal to the flow direction. Sea-level condition8 were assumed for the stagnation conditiona at the compressor entrance in all cases. Entrance conditions of compressors I, II, and III were assumed the same with an absolute entrance Mach number of 0.8 and no turning in the guide vanes. For compressor IV, an axial entrance Mach number of 0.39 and a turning of 30° were assumed in the guide vanes 80 that subsonic relative Mach numbers are obtained for rotational speeds up to 1300 feet per second.

When the air is decelerated through sonic velocity, the supersonic flow is assumed to be isentropically compressed to the minimum area that will permit stable supersonic flow entry (reference 1) and a normal shock is assumed to occur in this minimum passage area at the local Mach number. Losses in total pressure due to normal shock are included in the computation but losses due to friction, mixing, and other sources are neglected.

Computations for each compressor were made to determine the maximum theoretical total-pressure ratio available for a range of rotational speeds and rotor passage turning.

All symbols used are defined in appendix A.

Compressor I

A typical velocity diagram of compressor I (rotor-contained normal shock, subsonic stator) is shown in figure 1(a). Blade rotational speeds from 1200 to 1800 feet per second and varying values of the rotor-passage turning angle from 0° to 30° were assumed. The passage-area relations are based on a two-dimensional section without change in root or tip radius. The computational procedure for compressor I is given us an example in appendix B.

<u>Total-pressure ratio.</u> - The region of operation of compressor I is shown in figure 2(a) below the line $M_5 = 1.00$, where M_5 is the Mach number entering the stators. The maximum total-pressure ratio that can be obtained with speeds up to 1800 feet per second is 6.4 for one stage.

Very little turning in the rotor passage is required for total-pressure ratios of about 2, but the stator angles β_5 probably require two stator rows if the maximum turning that can be accomplished in one stator row is assumed to be about 40°. For total-pressure ratios of about 3.5, compressor I still has only a small turning (less than 10°) in the rotor for blade speeds above 1400 feet per second; however, the stator-entrance angles become rather high (about 70°).

Because of the present practical limitations on the operating parameters, only part of the area under the line $M_5 = 1.00$ (fig. 2(a)) appears available for actual compressor operation. The amount of diffusion possible in the rotor passages for blade solidities of from 1.50 to 3.50 is limited because at' the adverse conditions following the normal shook. Efficient diffusion to a Mach number as low as 0.45 may be assumed possible with these blade solidities if careful attention is paid to the pressure gradients caused by radial flows and to the blade design.

Mach number limitations *at* the diffusing stator entrance may also restrict the practical range of total-pressure ratios. Because of the large turning required, even with a force-break *Mach* number (reference 5) instead of the critical Mach number into the stator blades, a maximum absolute Mach number above 0.85 after the rotor cannot be expected to give good performance. The limitations on the angle and the Mach number into the stators may possibly be relaxed through application of the principle of the vaneless diffuser. Through conservation of moment of momentum, the tangential velocity and hence Mach number can be decreased by increasing the radius between the rotor and the stator. Similar considerations can also decrease the stator angle.

The flow angle8 at *the* stator inlet β_5 may become troublesome at the higher pressure ratios. A limit on the practical stator angle based on present conceptions may be assumed to exist at about 75°.

If the stator-entrance Mach number and rotor-exit Mach number (baaed on diffuser limitations) are assumed to be Limited to 0.85 and 0.45, respectively, the region of practicable performance is restricted to the double-shaded region in the lower left-hand part *of* figure Z(a). The maximum possible total-pressure ratio within the preceding assumed restrictions is then about 4.6 for rotational speeds up to 1700 feet per second.

The preceding considerations were based entirely on two-dimensional calculation8 with no change in root or tip radiue throughout the rotor or stator. If the root or tip radius is changed in order to restrict the exit annulus, the limitation on turning imposed by too rapid a diffusion can be relaxed. The degree to which the flow angle into the stators is improved when the maximum rotor diffusion is restricted to $M'_5 = 0.60$ is shown in figure 2(b) for a rotational, speed of 1600 feet per second. This curve can be compared with the curve in figure 2(a) *for* the same rotational speed computed without variation of root or tip radius. The lower total-pressure ratio for a given rotor turning due to the restricted rotor diffusion is quite apparent. In order to obtain a pressure' ratio in the restricted passage the same as that in the unrestricted passage, more turning is necessary to compensate for the lack of diffusion by adding kinetic energy. The use of the principle of the vaneless diffuser may also aid in relaxing the limitations on the maximum total-pressure ratio available from compressor I.

Compressor II

A velocity diagram for compressor II is shown in figure 1(b). The computations are the same as those of compressor I with an additional correction for deceleration through sonic velocity in the stators. Again, as for compressor I, the calculations assumed no change in root or tip radius.

Total-pressure ratio. - The theoretical total-pressure ratio available with compressor II is shown in figure 2(a) above the line $M_5 = 1.00$. For design operation with a shock in the rotor and sufficient kinetic energy to give supersonic velocities into the stators, the minimum total-pressure ratio is about 4.6. The maximum theoretical total-pressure ratio for blade speeds up to 1800 feet per second and 30° turning in the rotor passage is approximately 8.8. The required diffusion rate, however, is quite high for blades of about 4.0-inch chord length and the stator-entrance angles are so high that their accommodation is probably very difficult.

In order to avoid extreme rotor diffusion rates, high stator-entrance angles and stator-entrance Mach numbers only slightly above 1.00, radial restriction of the flow area at the rotor exit is probably necessary. The case for which the exit Mach number relative to the blade is held at 0.60 by radial restriction is shown for 1600 feet per second in figure 2(b) above the point $M_5 = 1.0$.

For a pressure ratio of 5, a turning angle in the rotor passage of 33° is required, with a Mach number into the stator of 1.1. With pressure ratios of about 6.5, the Mach number into the stator is about 1.28 with a turning angle of 51° required in the rotor passage. The angles into the stators are not prohibitively large but require at least two rows of stator blades to return the flow to the axial direction.

For proper operating conditions, compressor II must have a normal shock stabilized in both the rotor and the stator. A method must therefore be found for stabilizing two or more successive normal ehocks in the compressor passages in series by either exerting baok pressure on the shock farthest downstream or by using independent baok pressure on each rotor and stator row containing a shook. Difficulty may also arise if the stator angles differ sufficiently from the flow angle to cause too high a baok pressure to be exerted. on the compressor, thereby prohibiting the start of supersonic flow into the rotor.

Compressor III

Compressor III is differentiated from compressor II by the existence of supersonic flow throughout the rotor passage. A typical velocity diagram for compressor III is shown in figure 1(c) and the performance curve in figure 3. The relative Mach number at the rotor exit is assumed equal to the relative Mach number at the entrance (that is, all the energy added is kinetic) and it is also assumed that the flow is turned isentropically. The losses incurred by decelerating the air through the sonic velocity in the stators are included in the computation.

Total-pressure ratio. - The theoretical total-pressure ratio shown in figure 3 continuously increases with an increase in turning angle and rotational speed. For total-pressure ratios of about 2.0, the rotor-passage turning and stator-entrance angles (22.4° and 18.0°, respectively, for 1600 ft/sec) should offer no difficulty in design or operation. Total-pressure ratios of about 3.5 require rotor-passage turning of 40.8° for rotational speeds of 1600 feet per second, but the stator-entrance angle of 29° can still be handled in one row of stators. For average total-pressure ratios above 5.0, higher tip speeds may be desirable and two rows of stators my become necessary to return the flow to the axial direction. Stator-entrance Mach numbers above 2.0 lead to large losses through the normal shock (above 20 percent using the optimum contraction ratio). For total-pressure ratios of about 6.5, approach to isentropic deceleration through sonic velocity is essential for reasonably good efficiencies. Inasmuch as the normal shock occurs in the stator blades, however, this type of compressor may lend itself to the application of devices to accomplish isentropic deceleration through sonic velocity.

If it is assumed that a method may be found for maintaining the stator-shook losses to values below 20 percent, the principal limitation on compressor III is the required turning angle in the rotor passage. The restrictions on the amount and rate of turning possible in the rotor are not definitely known, but these restrictions are probably less severe than with compressors I and II, inasmuch as there is no normal shook to thicken the boundary layer and pressures

1067 along the blade can be controlled through proper cancellation of oblique shock waves to give a minimum of adverse pressure gradients. The total-pressure ratio available when isentropic deceleration can be obtained is shown by the upper curve of figure 3 for a tip rotational speed of 1800 feet per second. ($\Theta = 90°$ can be accomplished with ave. 10% loss in total pressure for $M_2 = 1.70$ – see Liccini & Ferri reports)

Compressor IV

Compressor IV utilizes a subsonic blower that imparts sufficient kinetic energy to the flow to give supersonic flow into the stators. A typical velocity diagram is shown in figure l(d). A prerotation of 30° in the inlet guide vanes and an absolute Mach number entering the rotor were assumed so that the rotor relative entrance velocity remains subsonic for rotational speeds up to 1300 feet per second. The flow in the rotor is assumed to be turned isentropically with the relative Mach number at the exit the same 88 the relative entrance Mach number.

<u>Total-pressure ratio.</u> - The maximum total-pressure ratio theoretically possible is shown in figure 4. For a total-pressure ratio of 2.0, a rotor passage turning of 43° and a stator-entrance angle of 42° are required at a rotational speed of 1200 feet per second. Based on these values, a pressure ratio Of 2.0 appears readily attainable with good efficiency. At pressure ratios of 3.5, a rather high turning angle of 78° is required in the rotor passage at 1200 feet per second. One of the limitations on this compressor is the amount of turning feasible in the rotor passage. If turning up to 90° can be obtained, theoretical total-pressure ratios above 4.5 per stag8 are possible.

Comparative Results

<u>Efficiency.</u> - The comparative loss in total pressure through the shock at a rotational speed of 1600 feet per second for compressors I, II, and III and at a rotational speed of 1300 feet per second for compressor IV plotted against total-pressure ratio is shown in figure 5. The losses are less than 10 percent at all pressure ratios considered for compressors I, II, and IV. The shock losses for compressor III may be somewhat greater. Because the shook is looated in the stators for compressor III, however, devices for approaching isentropio deceleration can probably be more easily applied (reference 6).

Mass flow. - Inasmuch as the entrance conditions for compressors I, II, and III are the same, the mass flow per Unit area of the annulus is the same. The high entrance Mach number of 0.8 permits about 96 percent of the maximum mass flow through the annulus. The 1067 hub-tip ratio is restricted by the requirement of supersonic velocities relative to the rotor blade at the root, but the total mass flow from compressors I, II, and III equals the best subsonic compressors having an equivalent frontal area. The low entrance Mach number of compressor IV restricts the mass flow per unit area of the annulus. The axial Mach number used of 0.39 gives a mass flow of only 60 percent of the maximum through the annulus. Inasmuch as the restriction of always having supersonic velocities relative to the rotor root does not apply to compressor IV, larger hub-tip ratios may possibly be used and the total mass flow for a given frontal area may be comparable with that of compressors I, II, and III.

SUMMARIZING REMARKS

Analytical performance computations for four different supersonic-compressor configurations gave the following results:

Compressor I with a rotor-contained normal shock and subsonic velocity at the stator entrance offered a total-pressure ratio of about 3.5 per stage, and for no change in outer shroud radius was limited by the etator-entrance angles and stator-entrance Mach numbers to a maximum total-pressure ratio per stage of 4.6.

Compressor II had a rotor-contained normal shock but permitted supersonic velocities at the stator entrance. Total-pressure ratios above 6.0 per stage were theoretically possible, but problems such as stabilizing normal shock8 in both the rotor and stator, large turning in the rotor passage, and utilizing the high stator-entrance angles require further investigation.

Compressor III utilized supersonic flow throughout the rotor and recovered the pressures in supersonic stators having a contained normal shock. Total-pressure ratios above 5.0 were possible in a single stage. [illegible]

Compressor IV had subsonic velocity in the rotor and supersonic velocity in the stator entrance. Theoretical total-pressure ratios above 4.0 per stage are possible, but the range of efficient supersonic operation was expected to be from 1.8 to 2.2.

The following table indicates the performance range for compressor types I, II, and III at an axial entrance Mach number of 0.80 and for compressor IV at an axial. entrance Mach number of 0.39:

Total-pressure ratio	Compressor \ Speed (ft/sec)	1000		1200		1400		1600		1800	
		θ[a] (deg)	β_5[b] (deg)	θ (deg)	β_5 (deg)	θ (deg)	β_5 (deg)	θ (deg)	β_5 (deg)	θ (deg)	β_5 (deg)
2	I			1.4	55.0	-0.4	54.0				
	III			30.6	23.0	26.4	20.0	22.4	18.0	19.4	16.0
	IV	56.0	47.0	43.0	42.0						
3.5	I					10.0	70.0	-1.6	69.0		
	III			56.0	40.0	47.0	33.0	40.6	29.0	36.2	26.0
	IV			79.0	54.2						
5.0	II							10.0	75.5	-1.5	75.5
	III							32.0	61.0		
						63.0	43.0	55.0	37.0	48.0	33.0
6.5	II							51.0	63.0		
	III									59.0	39.0

[a]Turning angle in rotor blade passages.
[b]Angle between absolute air velocity and compressor axis at stator entrance.

The modified compressor II, in which the diffusion at the rotor exit is restricted to $M'_5 = 0.60$, is used in this table because the amount of diffusion which would be required without change in rotor exit annulus is believed to be very difficult to obtain.

The viscous losses in each compressor were probably larger than those for an axial-flow subsonic rotor of the same pressure ratio. The shock losses at a speed of 1600 feet per

second for compressors I, II, III and at a speed of 1300 feet per second for compressor IV were less than 10 percent except for total-pressure ratios above 3.5 with compressor III. Because the shock was located in the stators for compressor III, however, devices far 1067
approaching isentropic deceleration may be more easily applied.

The specific mass flow of compressors I, II, and III for most designs Should be above 90 percent of maximum for the annular area. Compressor IV had a specific mass flow only about 60 percent of maximum, but because of the smaller radius ratio possible, may have a total mass flow comparable with the other types for a given frontal area.

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APPENDIX A

SYMBOLS

The following symbols are used in this analysis:

A area (consistent units)

a velocity of sound, ft/sec

C_R passage contraction ratio, $A_{inlet}/A_{minimum}$

E_R passage expansion ratio, $A_{minimum}/A_{exit}$

M absolute Mach number

M' Mach number relative to rotor blades

P absolute total pressure, lb/sq ft

P' relative total pressure, lb/sq ft

p static, or stream, pressure, lb/sq ft

T absolute total temperature, $^{\circ}R$

T' relative total temperature, $^{\circ}R$

t statio, or stream, temperature, $^{\circ}R$

U rotational speed of blade at radius r, ft/sec

V absolute velocity of air, ft/sec

V' velocity of air relative to rotor blades, ft/sec

β angle between absolute air velocity and compressor axis deg

β' angle between relative air velocity and compressor axis, deg

γ ratio of specific heats

θ turning angle in rotor blade passages, deg

Subscripts:

cr critical

min minimum

z axial direction

θ tangential component

0 stagnation condition

2 exit from guide vanes

3 immediately before shock in rotor

4 immediately after ehook in rotor

5 exit from rotor, entrance to stators

APPENDIX B

SAMPLE COMPUTATION

A sample computation is given for the performance characteristics of compressor I at a rotational speed of 1600 feet per second and a rotor-passage turning angle of 10°. These computations are based on one-dimensional relations, and tables or charts such as those of reference 7 greatly facilitate the computational process.

The following equations, which may be found in any book of supersonic flow (for example, reference 8), are used in the sample computation:

$$\frac{t}{T} = \frac{2}{2 + (\gamma-1)\, M^2} \tag{1}$$

$$\frac{p}{P} = \left(1 + \frac{\gamma-1}{2} M^2\right)^{-\frac{\gamma}{\gamma-1}} \tag{2}$$

$$\frac{A_{cr}}{A} = \frac{1}{M}\left[\frac{2}{\gamma+1}\left(1 + \frac{\gamma-1}{2} M^2\right)\right]^{\frac{\gamma+1}{2(\gamma-1)}} \tag{3}$$

$$\frac{P_4}{P_3} = \frac{\left(\frac{\gamma+1}{\gamma-1}\right)^{\frac{\gamma+1}{\gamma-1}} M_3^{\frac{2\gamma}{\gamma-1}}}{\left(\frac{2}{\gamma-1} + M_3^2\right)^{\frac{\gamma}{\gamma-1}} \left(\frac{2\gamma}{\gamma-1} M_3^2 - 1\right)^{\frac{1}{\gamma-1}}} \tag{4}$$

$$M_4^2 = \frac{1 + \frac{\gamma-1}{2} M_3^2}{\gamma M_3^2 - \frac{\gamma-1}{2}} \tag{5}$$

The following conditions are assumed:

$$M_{z,2} = 0.80 \qquad T_0 = 520^\circ$$

$$U = 1600 \text{ ft/sec} \qquad \theta = 10^\circ$$

Then from equation (1),

$$\left(\frac{t}{T}\right)_2 = 0.8865, \quad t_2 = 520\left(\frac{t}{T}\right)_2 = 460.0^\circ$$

$$a_2 = 49.1 \sqrt{t_2} = 1052$$

$$V_{z,2} = M_{z,2}\, a_2 = 841.9$$

From equation (2),

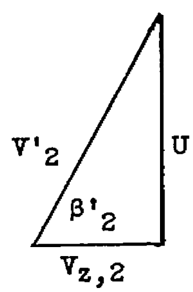
$$\left(\frac{p}{P_0}\right)_2 = 0.656$$

In accordance with the assumptions and the- velocity triangle,

$$\beta'_2 = \tan^{-1} \frac{U}{V_{z,2}} = 62.24^\circ$$

$$V'_2 = \frac{V_{z,2}}{\cos \beta'} = 1808 \text{ ft/sec}$$

$$M'_2 = \frac{V'_2}{a_2} = 1.719$$



From equation (1),

$$\left(\frac{t}{T'}\right)_2 = 0.6286$$

From equation (2),

$$\left(\frac{p}{P'}\right)_2 = 0.1969$$

$$T'_2 = \frac{t_2}{\left(\frac{t}{T'}\right)_2} = 733.4^\circ, \quad \frac{P'_2}{P_0} = \frac{0.656}{\left(\frac{p}{P'}\right)_2} = 3.331$$

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In order to determine the Mach number at the blade minimum section, the contraction-ratio curves for M'_2 given in figure 3 of reference 4 are used to find $C_R = 1.148$.

From equation (3) and the value of M'_2, $\left(\frac{A_{cr}}{A}\right)_{M'_2} = 0.7376$

Then

$$\left(\frac{A_{cr}}{A}\right)_{3,\ compressed} = C_R \left(\frac{A_{cr}}{A}\right)_{2,M'_2} = 0.8466$$

From equation (3) and $\left(\frac{A_{cr}}{A}\right)_{3,\ compressed}$,

$$M'_3 = 1.51$$

The total-pressure ratio across ths normal shook can be obtained from equation (4) for M'_3 as

$$\frac{P'_4}{P'_3} = 0.9266$$

from equation (5) for M'_3,

$$M'_4 = 0.6976$$

From equation (3) for M'_4, $\left(\frac{A_{cr}}{A}\right)_4 = 0.9123$

The expansion ratio between passage minimum area and exit is

$$E_R = C_R \frac{(\cos\ \beta')\ exit}{(\cos\ \beta')\ entrance} = 1.148 \times \frac{\cos\ 52.24}{\cos\ 62.24} = 1.510$$

Then

$$\left(\frac{A_{cr}}{A}\right)_5 = \frac{\left(\frac{A_{cr}}{A}\right)_4}{E_R} = 0.6042$$

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For $\left(\frac{A_{cr}}{A}\right)_5$ and equation (3), $M'_5 = 0.38$. From equations (1) and (2) for M'_5,

$$\left(\frac{t}{T'}\right)_5 = 0.9719, \quad \left(\frac{p}{P'}\right)_5 = 0.9052$$

$$t_5 = T'_2 \left(\frac{t}{T'}\right)_5 = 712.5'$$

$$\frac{P'4}{P_0} = \frac{P'_2}{P_0}\,\frac{P'_4}{P'_3} = 3.088$$

$$\frac{p_5}{P_0} = \frac{P'4}{P0}\left(\frac{p}{P'}\right)_5 = 2.793$$

$$a_5 = 49.1\sqrt{t_5} = 1311$$

$$V'_5 = M'_5\, a5 = 498.2$$

$$\beta'_5 = \beta_2 - \theta = 52.24^{\circ}$$

$$V_{z,5} = V'_5 \cos \beta'_5 = 305.1$$

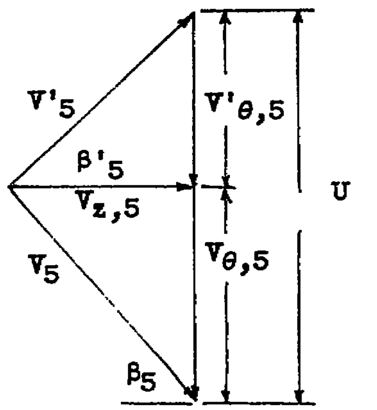
$$V'_{\theta,5} = V'_5 \sin \beta'_5 = 394$$

$$V_{\theta,5} = U - V'_{\theta,5} = 1206$$

$$85 = \tan^{-1}\left(\frac{V_{\theta,5}}{V_{z,5}}\right) = 75.8^{\circ}$$

$$V_5 = \frac{V_{\theta,5}}{\sin 85} = 1244$$

$$M_5 = \frac{V_5}{a_5} = 0.949$$



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Frau equations (1) and (2) for M_5,

$$\left(\frac{t}{T}\right)_5 = 0.847$$

$$\left(\frac{p}{P}\right)_5 = 0.5602$$

Then

$$T_5 = \frac{t_5}{\left(\frac{t}{T}\right)_5} = 841.2^{\circ}$$

$$\frac{P_5}{P_0} = \frac{\frac{p_5}{P_0}}{\left(\frac{p}{P}\right)_5} = 4.986$$

The compressor shock losses are equal to the normal shock losses except for compressor II, which has tot81 shook losses equal to 1.00 minus the product of the rotor and the stator normal-shook recoveries.

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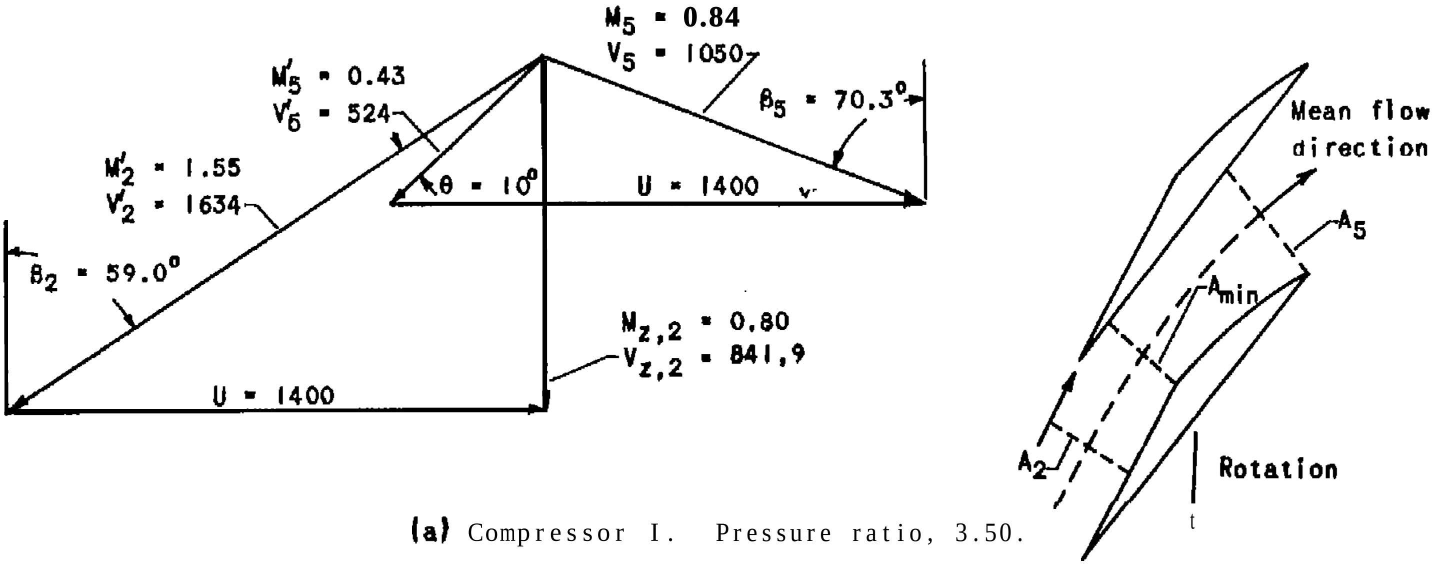
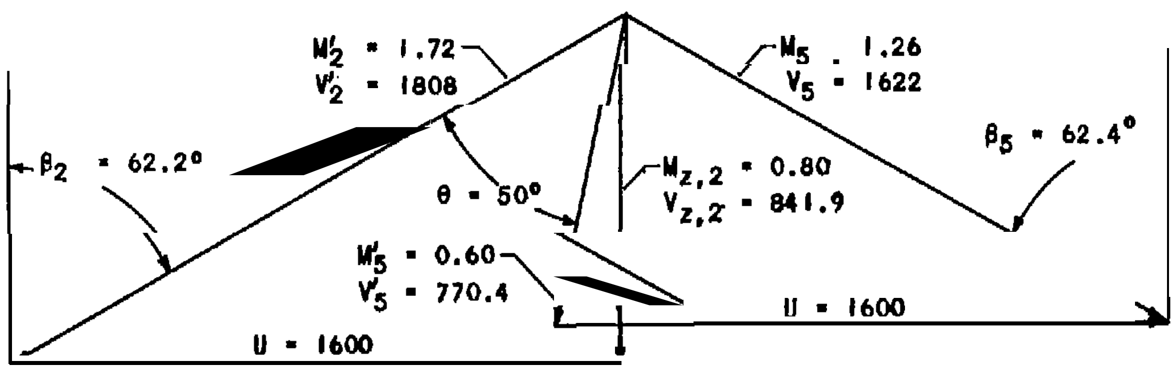
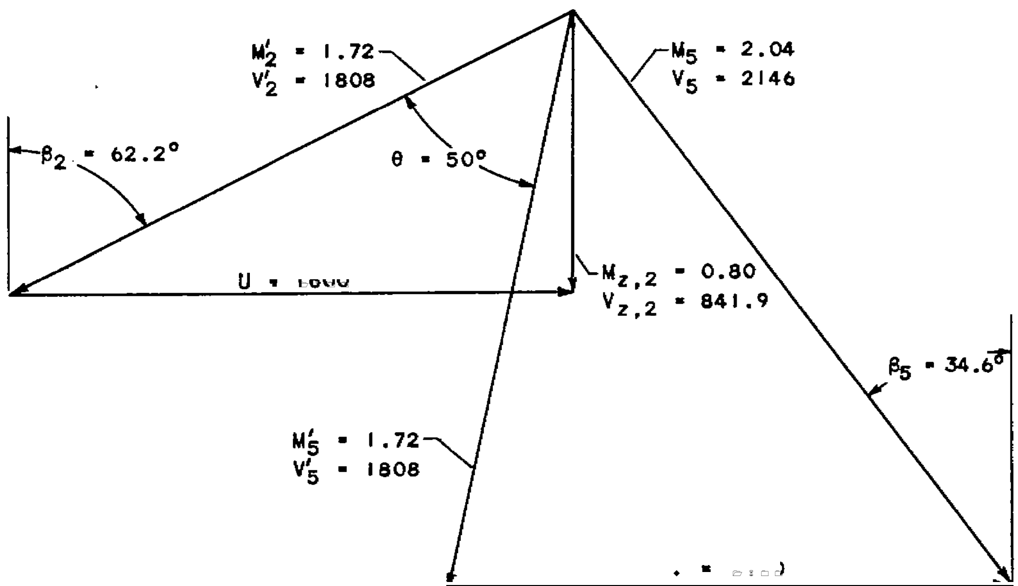


(a) Compressor I. Pressure ratio, 3.50.

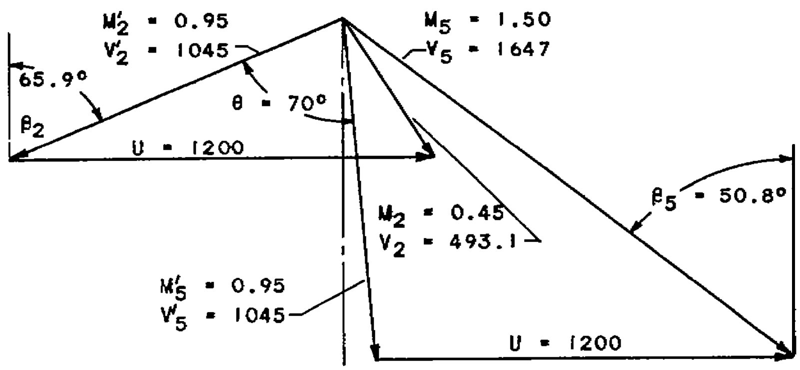


(b) Compressor II. Pressure ratio, 6.38.

Figure 1. - Representative velocity diagrams for supersonic compressor.



(c) Compressor III. Pressure ratio, 4.45.



(d) Compressor IV. Pressure ratio, 3.12.

Figure 1. - Concluded. Representative velocity diagrams for supersonic compressor.

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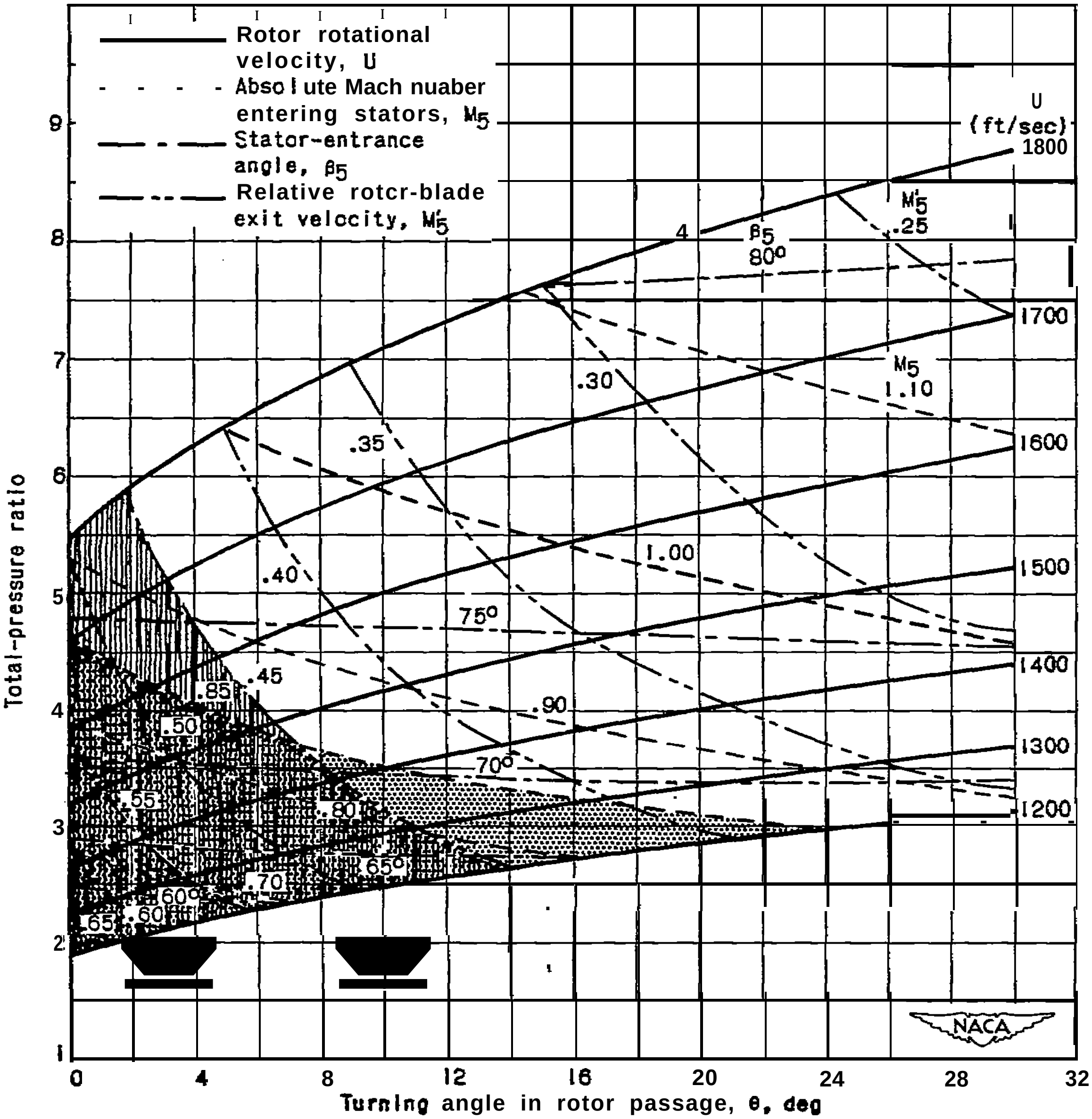

(a) Constant hub and tip radii through passage. Compressor I below line M_5 = 1.00; compressor II above line M_5 = 1.00.

Figure 2. - Over-all performance of supersonic compressors I and II. Specific mass flow, 0.96; $M_{z,2}$, 0.80.

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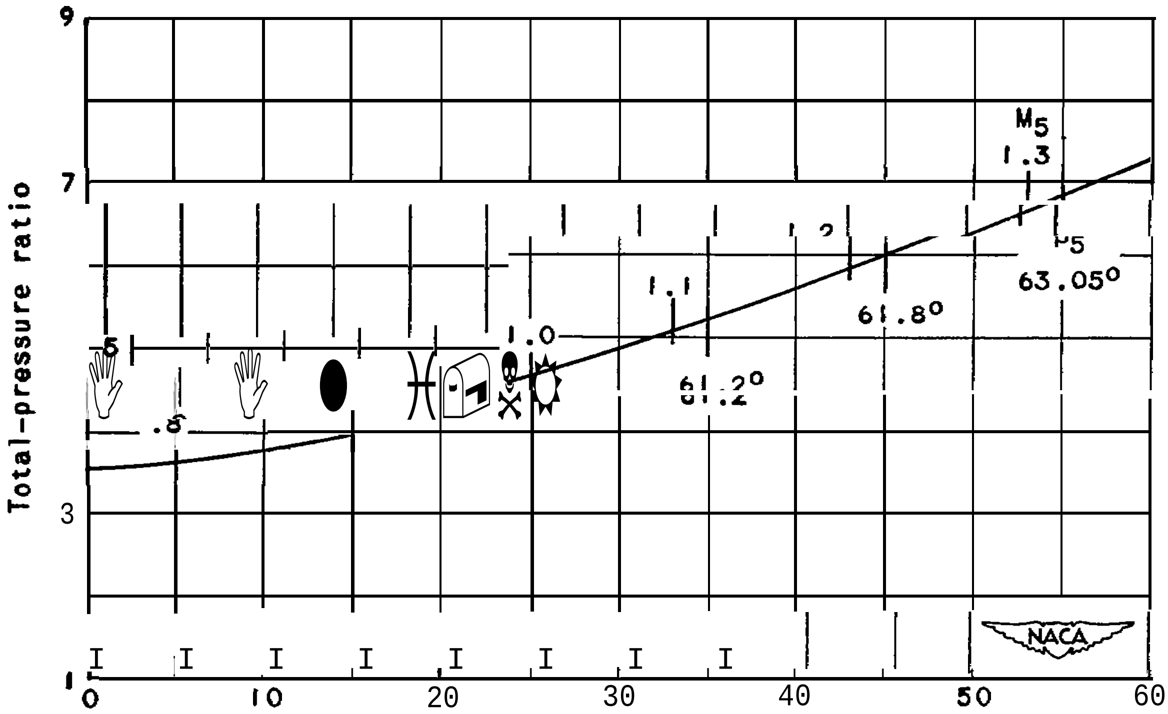

(b) Hub and tip tapered so that $M_5' = 0.60$; U, 1600 feet per second.

Figure 2. – Concluded. Over-all performance of supersonic compressors I and II. Specific mass flow, 0.96; $M_{z,2}$, 0.80.

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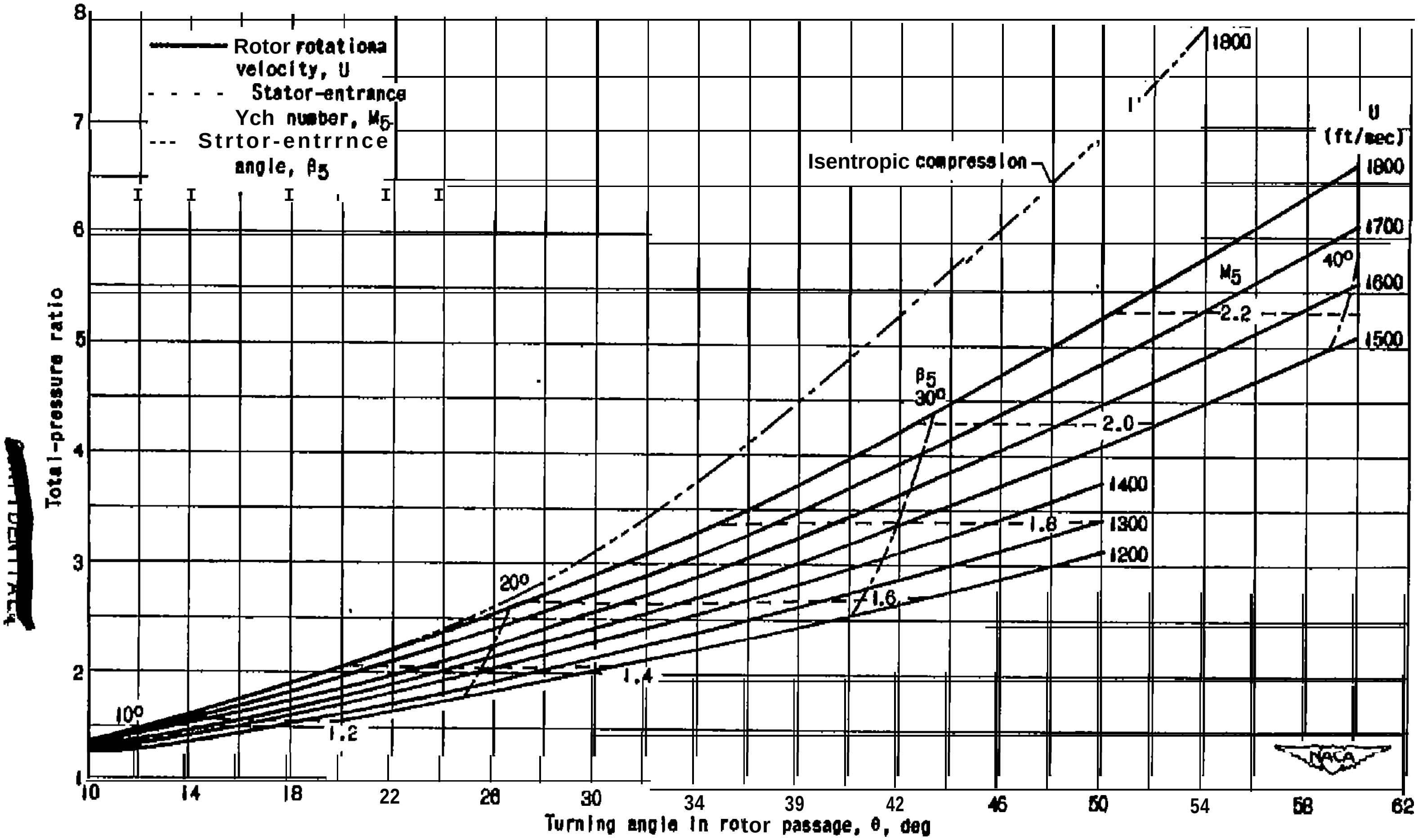

Figure 3. - Over-all performance of supersonic compressor III. $M_{z,2}$, 0.80; specific mass flow, 0.96.

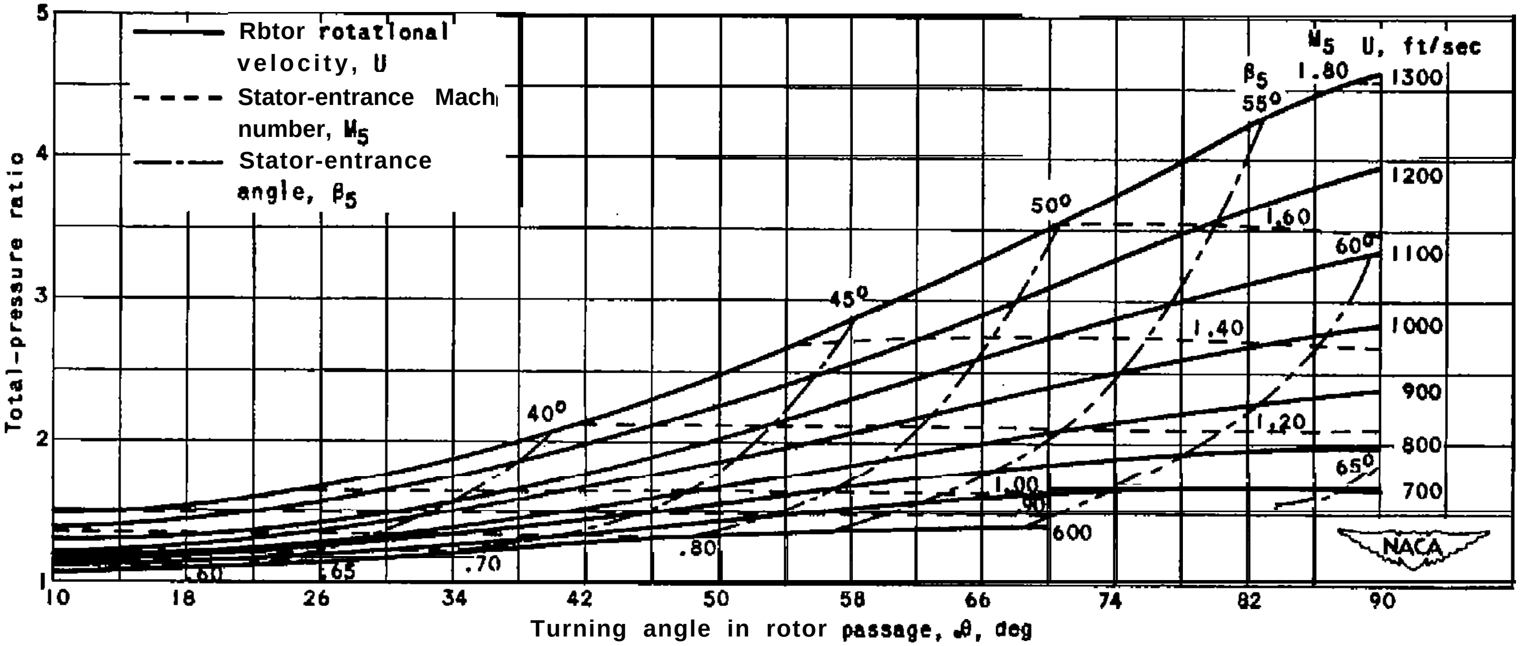

Figure 4. - Over-all performance ot supersonic compressor IV. $M_{z,2}$, 0.39: specitic mass floe. 0.616.

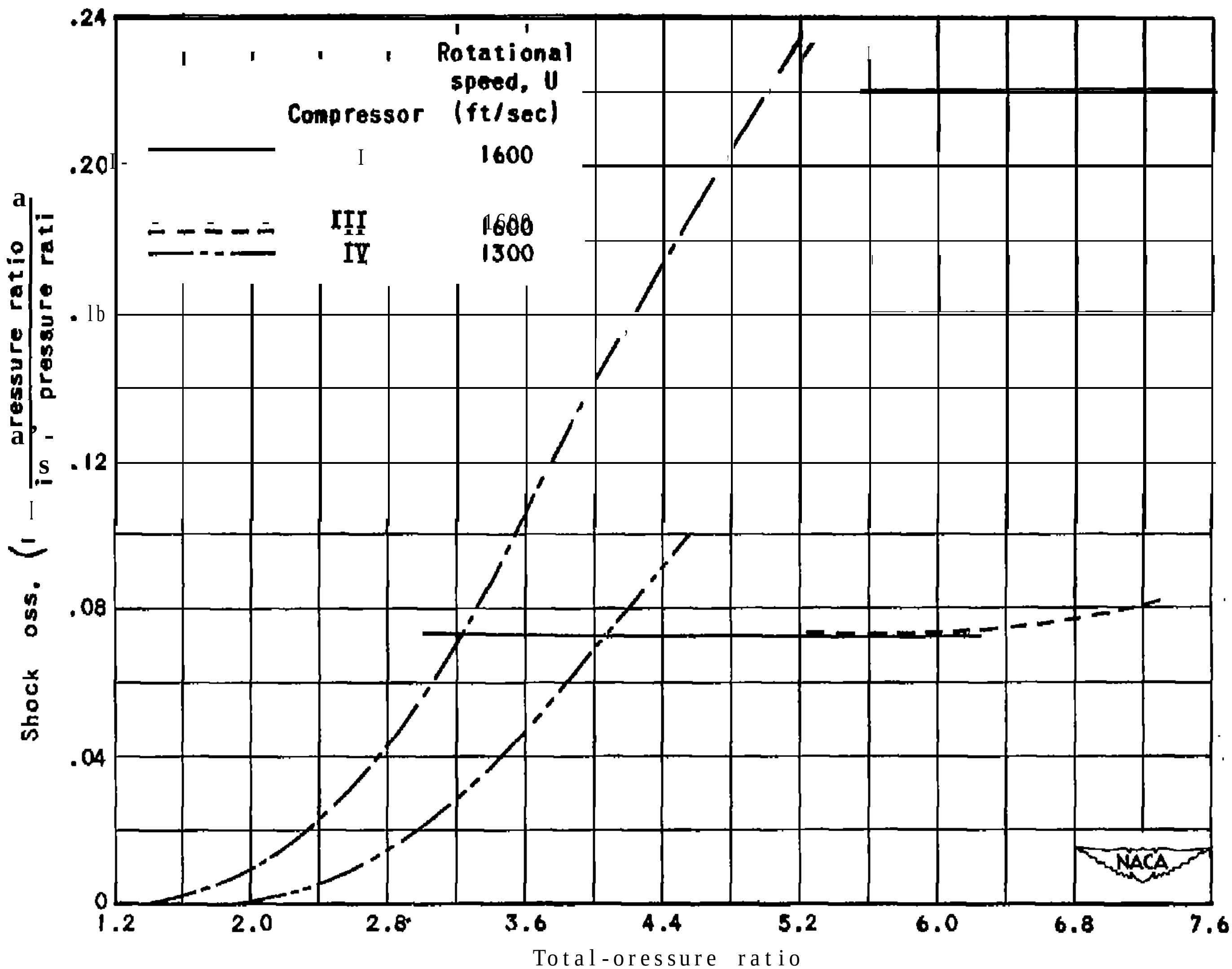

Figure 5. - Comparison of shock losses in supersonic compressors I, II, III, and IV.